Analytical Model Application for Adhesive Cylindrical Assemblies made by Hybrid Materials

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The present study makes a comparative study of various cylindrical assemblies using the analytical models developed by minimizing the potential energy associated with the stress field and using variational methods and some assumptions. All the components of the stress field were defined function of the $\sigma^{(1)}_{zz}(z)$ stress in the first element and then introduced into the potential energy formulation. A variational method applied on the potential energy of deformation help us to obtain the minimum. This applied model can predict the optimal overlap length. Finally, a failure criterion of Tsai-Hill type was established.

Keywords: stress analysis; stress distribution; adhesive joints; analytical method

The adhesively bonded joint lead to a good load distribution and is suitable for assemblies like composite / composite, composite / metal or metal / metal. This type of assemblies prevents damage caused by composite drilling, but a good bonding need to solve a problem of surfaces and interfaces. For this, a surface treatment must be applied to create a new surface without changing the mass properties of the substrate.

The mechanical performance of this assembly type is related to the distribution of the stresses in the adhesive layer [1, 2] which, because of its complexity, makes prediction of fractures difficult.

The first studies were carried out for plane assemblies with a simple overlap subject to tensile loads. Because the effects of bending of the adherents are not taken into account, a false evaluation of the level of maximum stress was related in 1938 in the work of Volkersen [3]. From this work to the more recent studies by the finite element method, many formulations have been proposed to define the stress field with increasing accuracy [4-18].

In the last decades many scientific publications concerning plane joints, with simple or double lap have been published but there are only a few theoretical works concerning the study of the mechanical behaviour in cylindrical adhesive assemblies [19-30]. A comparison between the results obtained experimental and by analytical computational methods, using the classical theory to which were added correcting fields, have been recently reported [31, 32].

The first stress field in a cylindrical joint subject to axial load was built by Shi and Cheng [20] using equilibrium equations, conditions of continuity of the stresses at the interfaces and equations of compatibility. After that they calculated the potential energy associated with the stress field and using the theorem of minimal complementary energy obtained a system of differential equations. Using this principle Kumar [33] provided a simple analytical framework to study the stress intensity and distribution in adhesive joint.

Our interest is to develop new analytical model for a fast pre-dimensioning of cylindrical or double-lap adhesive bonded assemblies. This paper presents a comparative study of various cylindrical assemblies using the analytical models developed by minimizing the potential energy associated with the stress field and using variational methods and some assumptions [1, 34-37]. The method used to obtain the optimal stress field for cylindrical joints consists of the following phases: construction of a stress field, calculation of the potential energy associated with this stress field, minimization of this energy by variational calculus, resolution of the differential equation obtained.

Cylindrical assembly definition

For this study we consider an assembly of tubes subject to a tensile load whose geometrical definitions are showed in figure 1.

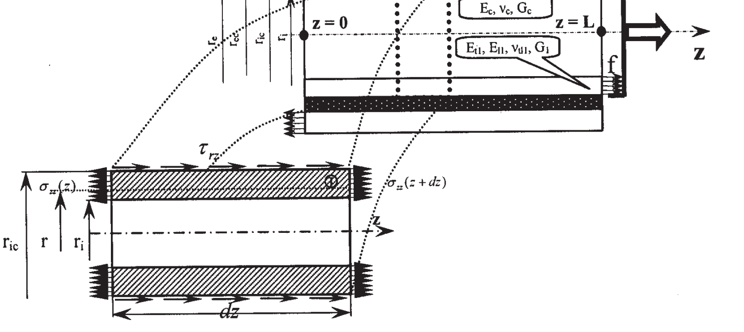

Fig. 1. Cylindrical assemblies

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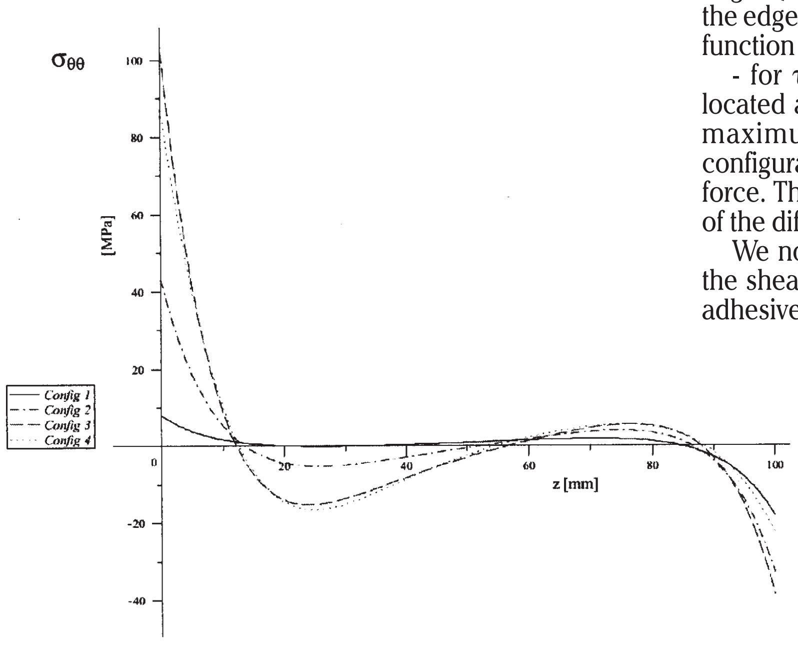
	Tube 1	Adhesive	Tube 2	r_i [mm]	r_{ic} [mm]	r_{ec} [mm]	r_e [mm]	L [mm]	f [MPa]
Config 1.	Titanium TA 6V E = 105000 MPa G = 40385 MPa υ = 0.3	Araldite Redux 340 SP E_c = 2700 MPa G_c = 1000 MPa υ_c = 0.35	Carbon/Epoxy 90°/± 17.2° E_x = 60730 MPa E_y = 100200 MPa G_{xy} = 9356 MPa υ = 0.07	44.8	47.8	48	54.5	100	100
Config 2.	Aluminium ALU 7075 E = 72000 MPa G = 27692 MPa υ = 0.3		Aluminium ALU 7075 E = 72000 MPa G = 27692 MPa υ = 0.3	44.8	47.8	48	51		
Config 3.	Aluminium ALU 7075 E = 72000 MPa G = 27692 MPa υ = 0.3		Carbon/Epoxy G0969/M18 E_x = 44080 MPa E_y = 44080 MPa G_{xy} = 16640 MPa υ_{xy} = 0.325	44.8	47.8	48	50		
Config 4.	Carbon/Epoxy ± 55° E_x = 11410 MPa E_y = 32600 MPa G_{xy} = 32680 MPa υ = 0.433		Carbon/ Epoxy ± 45° E_x = 17090 MPa E_y = 17090 MPa G_{xy} = 36380 MPa υ = 0.781	44.8	48.1	48.3	51.6		

Table 1
PARAMETERS OF THE CYLINDRICAL ASSEMBLY

We use the following notations: E_c, ν_c, Young's modulus and Poisson's ratio of the adhesive; E_{t1}, E_{l1}, ν_{tl1}, longitudinal, transverse modulus and Poisson's ratio of the inner tube; E_{t2}, E_{l2}, ν_{tl2}, longitudinal, transverse modulus and Poisson's ratio of the external tube; r_i, internal radius of the inner tube; r_{ic}, external radius of the inner tube; r_{ec}, internal radius of the external tube; r_e, external radius of the external tube; L, overlap length; f and q, tensile stresses along the z axis, on the inner and outer tubes, respectively.

The stresses in the various materials will be defined by the index (i), (i = 1 for the inner tube, c for the adhesive and 2 for the outer tube).

The application developed in this work is presented using the parameters from table 1 and the analytical model developed by the author [1, 36]. The basis of this analytical model was the analysis of the stresses applied to an elementary volume of the assembly under consideration, observing the boundary conditions, the geometry and materials of the assembly. The application of an energy method made it possible to obtain the solution of the problem in stress in any point of the structure. The behaviour law enabled us to obtain the deformations then, by integration, the displacements. The problem in stress, deformation and displacements was thus entirely defined.



a)

Fig. 2. Stress distributions in the adhesive: a. Orthoradial stress ($\sigma_{\theta\theta}$)

Thus the analytical model makes it possible to determine the rigidity of the assembly and to obtain a simple formulation very quickly, which gives the total behaviour of the assembly.

Model application and parametric study

The analytical model developed by minimizing the potential energy associated with the stress field will be used in the present study to analyse the influence of various parameters affecting the intensity and distribution of the stresses in cylindrical adhesive assemblies with tube elements. This analysis will be reduced to a study of the influence of the following parameters: materials type, adhesive type and thickness and overlap length. The assembly parameters are showed in table 1.

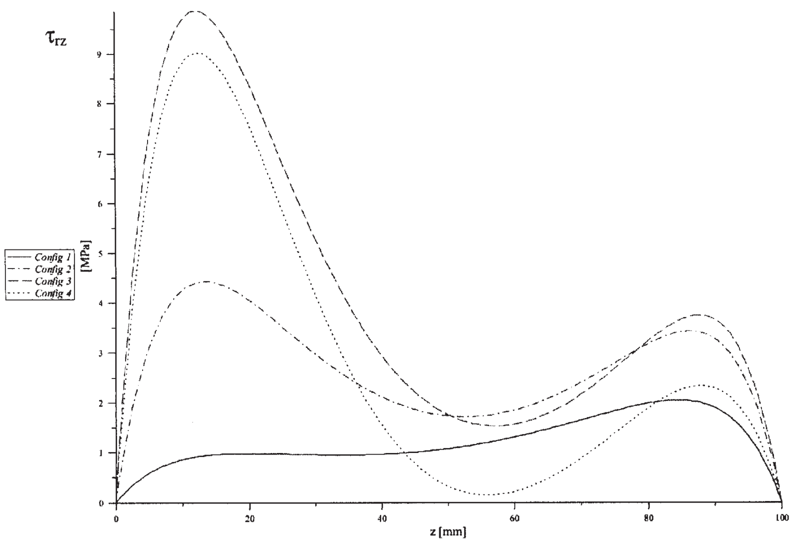
Stress distribution

Figure 2 shows the stress distributions in the adhesive for the analysed configurations, that is to say the distributions of the orthoradial and shear stresses.

The analysed configurations are showed in table 1. We notice that:

- for $\sigma_{\theta\theta}$, the maximum values are obtained on the free edges (z = 0, z = L) (fig. 2a) and they are very localized at the edges. The balance between the maximum values are function of materials configuration;
- for τ_{rz}, we observe (fig. 2b) two peaks of stresses located at equal distances from the two free edges. The maximum value is varying function of assembly configuration and it is between 1 and 10 % of the applied force. The peaks do not have the same intensity because of the difference of rigidities of the two bonded tubes.

We note that the orthoradial stresses are greater than the shear stresses. The use of a fracture criterion for the adhesive bonded joint must take into account not only the



b)

Fig. 2. Stress distributions in the adhesive
b) Shear stress (τ_{rz})

shear stress τ_{rz} but also the orthoradial stress $\sigma_{\theta\theta}$. We can establish a failure criterion of the Hill-Tsai type as follows:

$$K_T = \underbrace{\left(\frac{\sigma_{\theta\theta}^{(c)}}{\sigma_R^{(c)}}\right)^2}_{K_\sigma} + \underbrace{\left(\frac{\tau_{rz}^{(c)}}{\tau_R^{(c)}}\right)^2}_{K_\tau} \quad (1)$$

$$K_T \rightarrow \begin{cases} \geq 1 \text{ - failure} \\ < 1 \text{ - no failure} \end{cases} \quad (2)$$

It should be noted that taking the orthoradial stresses into account is of primary importance.

Influence of the overlap length

The influence of covering length on the shear stress distribution for configuration 3 Aluminium Al - Carbon/ Epoxy (table 1) is shown in figure 3.

We note that when the length of cover exceeds an "optimum length" a part of this overlap is redundant. The analysis of the shear stress distribution shows that after 180 mm we have only tow active zones equal and situated on the borders of the overlap. This makes it possible to determine a useful maximum length of overlap. By increasing the covering length gradually we observe the reduction of shear stress in the adhesive joint and the displacement of the peaks of stresses towards the free edges. We notice that there is an optimal length beyond which the maximum stresses do not evolve (fig. 4).

Influence of the adhesive rigidity

The influence of Young's modulus (E_c) of the adhesive on the shear stress is presented in figure 5.

We observe that the maximum peaks increase slightly when the elastic modulus increases. This analytical model, help us to decide the type of the adhesive function of his Young's modulus.

Influence of adhesive thickness

The influence of adhesive thickness (e_c) on the intensity and distribution of shear and orthoradial stresses is shown in figures 6 and 7.

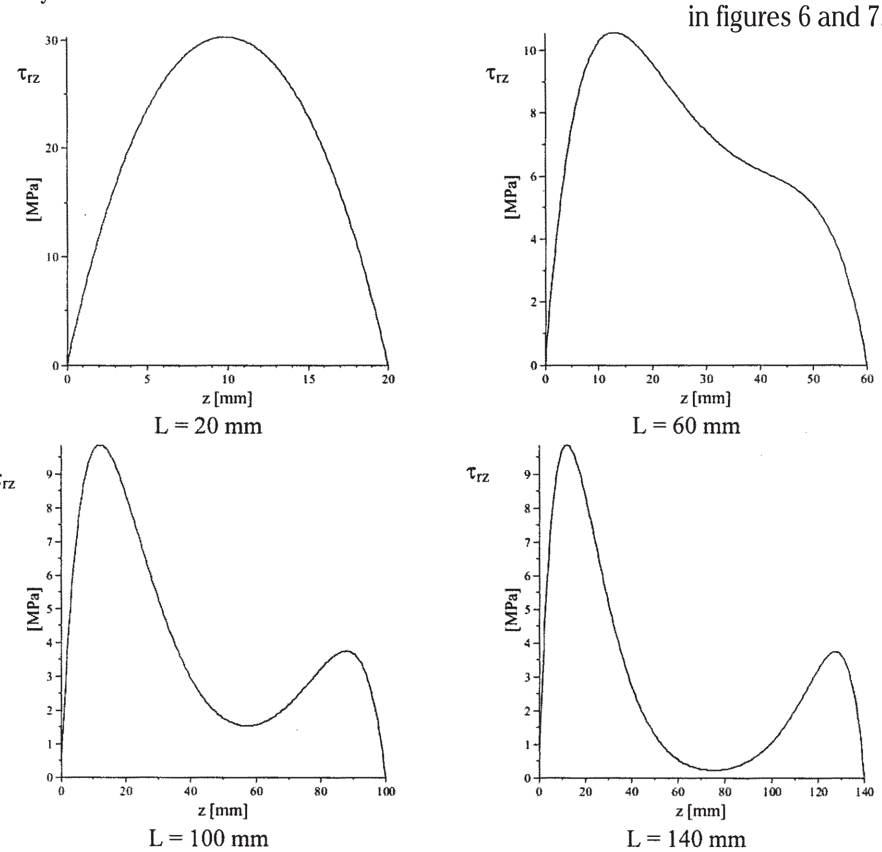


Fig. 3.

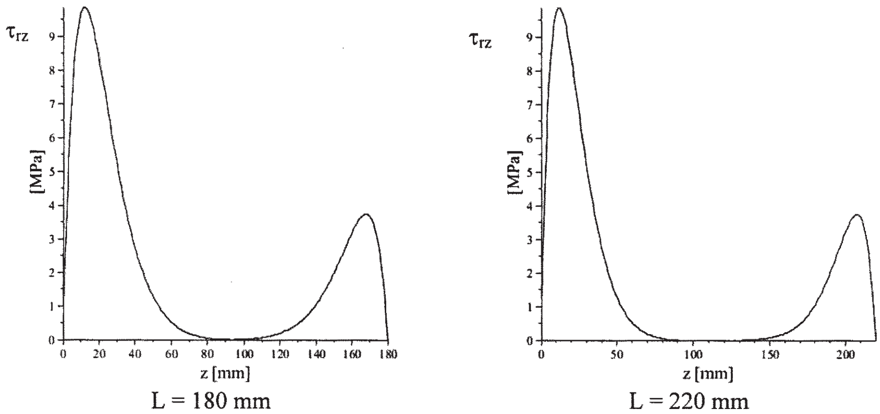


Fig. 3. Shear stress (τ_{rz}) distribution according to the overlap length for configuration 3 (Aluminium Al - Carbon/Epoxy)

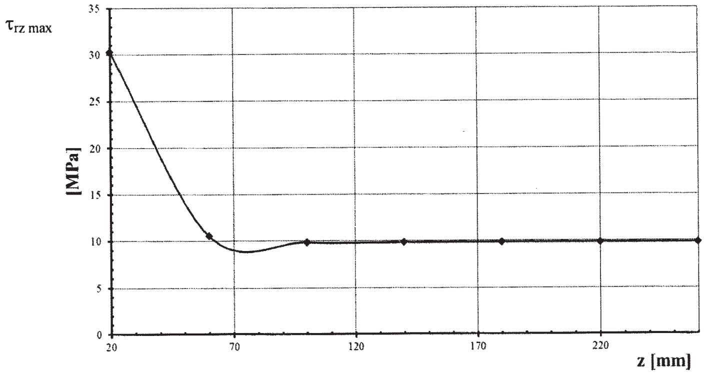


Fig. 4. Maximum shear stress ($\tau_{rz\ max}$) distribution according to the overlap length for configuration 3 (Aluminium Al - Carbon/Epoxy)

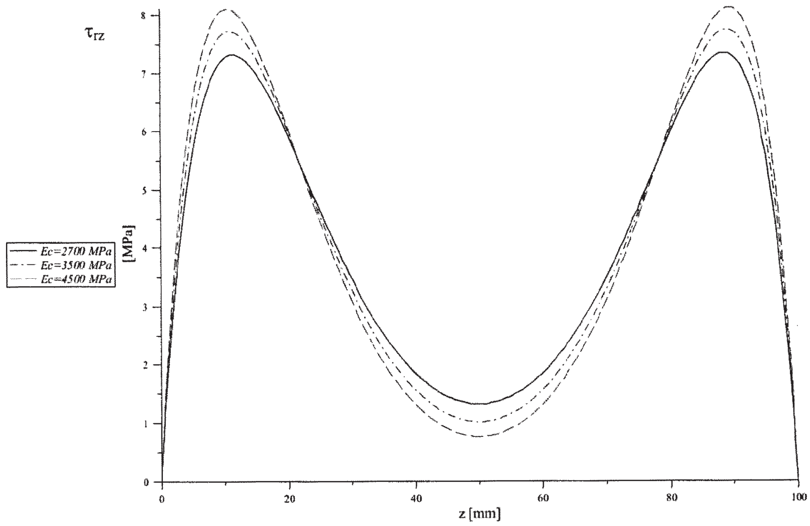


Fig. 5. Shear stress (τ_{rz}) variation according to Young's modulus of the adhesive length for configuration 3 (Aluminium Al - Carbon/Epoxy)

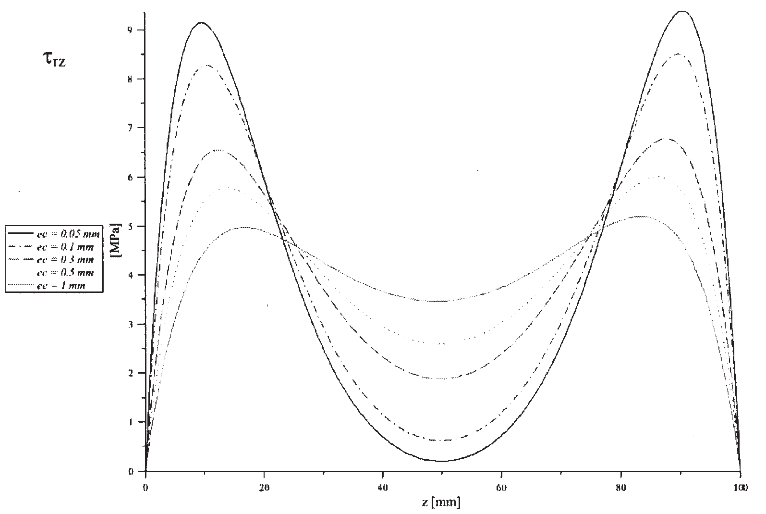


Fig. 6. Shear stress (τ_{rz}) variation according to adhesive thickness length for configuration 3 (Aluminium Al - Carbon/Epoxy)

We observe that as the thickness of adhesive increases, the values of the stresses decrease at the free edges. The distribution tends to become uniform.

After the analysis of the suggested configuration and the influence of geometrical and physical parameters on

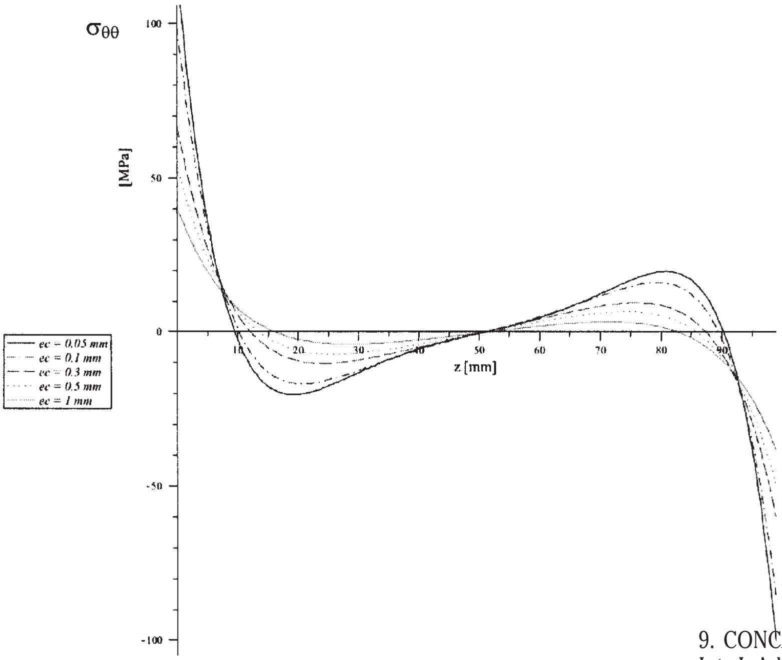


Fig. 7. Orthoradial stress ($\sigma_{\theta\theta}$) variation according to adhesive thickness length for configuration 3 (Aluminium Al - Carbon/Epoxy)

the stress field we can observe that bonded composite assemblies have the same behaviour as metal adhesive-bonded joints.

Conclusions

Adhesive joining is a simple method of assembly. Its interest lies in the fact that it minimizes the machining of the parts to be assembled. The performance of the adhesive bonded joints depends on the performance of the adhesive. The latest generations of adhesives, delivered in the form of film, make it possible to minimize the number of operations to make the join and greatly increase the mechanical resistance. However, the designing engineer must have at his disposal methods and/or reliable computer codes for pre-dimensioning with known limits.

The objective of this study was to analyse some cylindrical assemblies configurations using an analytical model for dimensioning adhesive-bonded joints.

The analysis was carried out on the distribution of stress in the substrates and the adhesive joint. The distribution of stress in the adhesive remained very close to the solution given by finite elements performed in research group [35]. The analytical model underestimated the stresses in the adhesive leading to an over-estimate of the forces at fracture. However, this model is reliable and allows fast analysis of this type of assembly.

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References

1. NEMEŞ, O., LACHAUD, F., MOJTABI, A., Int. J. Adhesion Adhesives, 26, nr. 6, 2006, p. 474.
2. NEMEŞ, O., LACHAUD, F., Int. J. Adhesion Adhesives, 30, nr. 5, 2010, p. 288.
3. VOLKERSEN, O., Luftfahrtforschung, 15, 1938, p. 41.
4. HART-SMITH, L. J., Adhesive-bonded single-lap joints, CR-112236, NASA, 1973.
5. BIGWOOD, D.A., CROCOMBE, A.D., Int. J. Adhesion Adhesives, 9, 1989, nr. 4, p. 229.
6. TSAI, M. Y., OPLINGER, D., MORTON, W. J., Int. J. Solids. Struct., 35, nr. 12, 1998, p. 1163.
7. LI, G., LEE-SULLIVAN, P., Int. J. Adhesion Adhesives, 21, nr.3, 2001, p. 211.
8. CHALKLEY, P., Int. J. Adhesion Adhesives, 21, nr.3, 2001, p. 241.
9. CONCAVALES, J.P.M., DE MOURA, M.F.S.F., DE CASTRO, P.M.S.T., Int. J. Adhesion Adhesives, 22, nr. 5, 2002, p. 357.
10. MORTENSEN, F., Thomsen, O. T., Composite Struct., 56, nr.2, 2002, p. 165.
11. MORTENSEN, F., Thomsen, O. T., Composites Sci. Technol., 62, nr. 7-8, 2002, p. 1011.
12. XIAO, X. R., FOSS, P. H., SCHROEDER, J. A., Int. J. Adhesion Adhesives, 24, nr.3, 2004, p. 229.
13. XIAO, X. R., FOSS, P. H., SCHROEDER, J. A., Int. J. Adhesion Adhesives, 24, nr.3, 2004, p. 239.
14. TUDOR, D.I., PARAUSAN, I. HADAR, A., Mat. Plast., **49**, no. 3, 2012, p. 166.
15. GROZEA, M.-A., HADAR, A., Mat. Plast., **48**, no. 2, 2011, p. 164.
16. GOANTA, V., HADAR, A., LEITOIU, B., Mater. Plast., 47, nr. 4, 2010, p. 450.
17. ILIESCU, N., HADAR, A., PASTRAMA, S.D., Mat. Plast., **46**, no. 1, 2009, p. 91.
18. PARAUSANU, I., HADAR, A., PARASCHIV G., VISAN, D.C., Mat. Plast., **50**, no. 2, 2013, p. 106.
19. ADAMS, R. D., PEPPIATT, N., J. Adhesion, 9, 1977, p. 1.
20. SHI, Y. P., CHENG, S., J. Eng. Mech. 119, nr.3, 1993, p. 584.
21. SERRANO, E., Int. J. Adhesion Adhesives, 21, nr. 2, 2001, p.115.
22. ÇOLAK, A., Int. J. Adhesion Adhesives, 21, nr. 6, 2001, p.487.
23. BAINBRIDGE, R., METTEM, C., HARVEY, K., ANSELL, M., Int. J. Adhesion Adhesives 22, nr. 1, 2002, p. 47.
24. KAWAMURA, H., SAWA, T., YONENO, M., NAKAMURA, T., Int. J. Adhesion Adhesives, 23, nr. 2, 2003, p.131.
25. LUBKIN, L., REISSNER, E., J. Appl .Mech., 78, 1956, p. 1213.
26. ALWAR, R. S., NAGARAJA, Y. R., J. Adhesion, 8, 1976, p. 76.
27. UPDIKE, D. P., YUCEOGLU, U., in: Proc. 4th International Conference on Composite Materials, Tokyo, 1982, p. 297.
28. THOMSEN, O. T., Composite Struct., 21, nr. 4, 1992, p. 249.
29. FROSTING, Y., THOMSEN, O. T., MORTENSEN, F., J. Eng. Mech., 125, nr. 11, 1999, p.1298.
30. VAZIRI, A., NAYEB-HASHEMI, H., Int. J. Adhesion Adhesives, 22, nr. 4, 2002, p. 367.
31. APALAK, M. K., GUNES, R., EROGLU, S., Int. J. Adhesion Adhesives, 27, nr. 1, 2007, p. 26.
32. DEB, A., MALVADE, I., BISWAS, P., SCHROEDER, J., Int. J. Adhesion Adhesives 28, nr.1, 2008, p. 1.
33. KUMAR, S., Int. J. Adhesion Adhesives 29, nr. 8, 2009, p. 785.
34. NEMES, O., Studia Universitatis Babes-Bolyai Chemia, 52, nr.4, 2007, p. 175.
35. NEMES, O., LACHAUD, F., MOJTABI, A., BORZAN, M., GRIGORAS, St., Mat. Plast. **45**, no. 4, 2008, p. 390.
36. NEMES, O., LACHAUD, F., J. Adhes Sci Technol, 23, nr. 10-11, 2009, p. 1383.
37. NEMES, O., Studia Universitatis Babes-Bolyai Chemia, 55, nr.2, 2010, p. 389.

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